

Exercises <sup>[A]</sup>

1. Write the domain and range of the functions defined below.  
 (a)  $f = \{(1, 4), (2, 9), (3, 19), (4, 51)\}$  (b)  $f(x) = x^3$  (c)  $f(x) = x^2 + 2$   
 (d)  $f(x) = \frac{6}{x-2}$  (e)  $f(x) = \sqrt{x^2 - 4}$
2. (a) Explain why  $y^2 = 9 - x^2$  does not define a function.  
 (b) Does  $y = \sqrt{9 - x^2}$  define a function?
3. State the domain of the function defined by each equation.  
 (a)  $y = \frac{4}{x(x-2)}$  (b)  $f(x) = \frac{x}{x^2-1}$  (c)  $y = \frac{2}{\sqrt{x-1}}$
4. If  $f(x) = 4\sqrt{x-1} + x + 5$ , find the least value of  $f$ .  
*Hint:* First find the domain of  $f$ .
5.  $f(x) = 4x - 3$ . Write the simplest form of the following expressions.  
 (a)  $f(3)$  (b)  $f(0)$  (c)  $f(-2)$  (d)  $f(n)$  (e)  $f(x^2)$   
 (f)  $f(n+1) - f(n)$  (g)  $\frac{f(a) - f(b)}{a - b}$  (h)  $\frac{f(x_1 + h) - f(x_1)}{h}$
6.  $f(x) = 9 - 3x$ . Write the simplest form of the following expressions.  
 (a)  $f(0)$  (b)  $f(4)$  (c)  $f(-3)$  (d)  $f(2k)$  (e)  $f(x+3)$   
 (f)  $f(n+1) - f(n)$  (g)  $\frac{f(a) - f(b)}{a - b}$  (h)  $\frac{f(x_1 + h) - f(x_1)}{h}$

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7.  $f(x) = x^2 - 4x^2$ . Show that  $f(-x_1) = f(x_1)$ .  
 (b) What can be deduced about the symmetry of the graph of  $f$ ?
8. (a)  $f(x) = x^3 - 4x$ . Show that  $f(-x_1) = -f(x_1)$ .  
 (b) What can be deduced about the symmetry of the graph of  $f$ ?
9. Which of the following statements are true for every function  $f$ ?  
 (a) The graph of  $f$  is symmetric about the  $y$ -axis.  
 (b) The graph of  $f$  is symmetric about the  $x$ -axis.  
 (c) If  $x_1 = x_2$ , then  $f(x_1) = f(x_2)$ .  
 (d) If  $f(x_1) = f(x_2)$ , then  $x_1 = x_2$ .  
 (e) A vertical line meets the graph of  $f$  in no more than one point.  
 (f) A horizontal line meets the graph of  $f$  in no more than one point.
10. Show that if a linear function is defined by  $f(x) = mx + k$ , the expression  $\frac{f(a) - f(b)}{a - b}$  is the slope of the graph of  $f$  when  $a, b$  are elements in the domain of  $f$  and  $a \neq b$ .
11.  $f(x) = x^2$ . Write the simplest form of the following expressions.  
 (a)  $f(3) - f(-3)$  (b)  $f(x_1) - f(-x_1)$   
 (c)  $f(a+b) - f(a-b)$  (d)  $\frac{f(x_1 + h) - f(x_1)}{h}$
12.  $f(x) = x^2$ . Evaluate  $\frac{f(a) - f(b)}{a - b}$ , where  $a \neq b$ , and give a geometric interpretation of the result.

## Exercises [A]

1.  $f(x) = 6 - 3x$ . (a) State the zero of  $f$ . (b) Find the solution set of  $f(x) > 0$ . (c) Find the change in  $f(x)$  when  $x$  is increased by 4.
2.  $f(x) = 2x + 5$ . (a) State the zero of  $f$ . (b) Find the solution set of  $f(x) > 0$ . (c) Find the change in  $f(x)$  when  $x$  is increased by 7.
3. A function  $F$  is defined by the rule  $y = 2x$  with the domain the set of positive integers. What is the range of  $F$ ?
4. A function  $g$  is defined by the rule  $y = \frac{1}{2}x$  with the range the set of positive integers. What is the domain of  $g$ ?
5.  $f(x) = 3x + 4$  and  $g(x) = 10 - x$ . (a) Find  $f \cap g$ . (b) Find the solution set of  $f(x) \geq g(x)$ .
6.  $f(x) = 3x + 4$  and  $g(x) = 3x + 8$ . (a) Find  $f \cap g$ . (b) Find the solution set of  $g(x) > f(x)$ .
7. The number of units of length of a metal bar is given by the formula  $l = 100.000 + 0.003(t - 55)$ , when  $t$  is the temperature on the Fahrenheit scale. (a) Find  $l$  when  $t = 55$ . (b) Find  $l$  when  $t = 0$ . (c) Find the least integer  $t$  for which  $l > 100.140$ . (d) Find the increase in  $l$  corresponding to an increase in temperature of  $20^\circ$ .
8. (a)  $f(x) = 12 - 3x$ . Evaluate  $f(4) - f(1)$ .  
(b)  $f(x) = 2.38x + 15.72$ . Evaluate  $f(17.5) - f(7.5)$ .  
(c)  $f(x) = mx + k$ . Write the simplest expression for  $f(n + 3) - f(n)$ .
9. (a) If  $f(x) = 2x - 5$ , evaluate  $f(a)$ ,  $f(b)$ ,  $f(a + b)$ . (b) Is it true that  $f(a + b) = f(a) + f(b)$ ?
10. (a) If  $f(x) = 4.2x$ , evaluate  $f(a)$ ,  $f(b)$ ,  $f(a + b)$ . (b) Is it true that  $f(a + b) = f(a) + f(b)$ ?
11. If  $f(x) = mx + k$ , under what condition does  $f(a + b) = f(a) + f(b)$  for all  $a$  and  $b$ ?
12. If  $F$  is a linear function which contains  $(2, -3)$  and  $(0, 1)$ , find  $F(x)$ .
13. If  $g$  is a linear function such that  $g(0) = 4$  and  $g(4) = 0$ , find  $g(t)$ .
14. If  $f$  is a linear function such that  $f(-2) = 4$  and  $\Delta f(x) = 3 \cdot \Delta x$ , find  $f(x)$ .
15. If  $f(t) = 3t - 2$ , find (a)  $f(2x)$ , (b)  $f(x^2)$ , (c)  $f(2t + 1)$ .
16. If  $f(t) = 3t - 2$ , find  $x$  in terms of  $t$  so that  $f(x) = t$ .

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1. Domain (a)  $\{1, 2, 3, 4\}$  (b)  $\{x: x \in R\}$  (c)  $\{x: x \in R\}$   
 (d)  $\{x: x \neq 2, x \in R\}$  (e)  $\{x: |x| \leq 2, x \in R\}$   
 Range (a)  $\{4, 9, 19, 51\}$  (b)  $\{f(x): f(x) \in R\}$   
 (c)  $\{f(x): f(x) \geq 2, f(x) \in R\}$  (d)  $\{f(x): f(x) \neq 0, f(x) \in R\}$   
 (e)  $\{f(x): f(x) \geq 0, f(x) \in R\}$
3. (a)  $\{x: x \neq 0, x \neq 2, x \in R\}$  (b)  $\{x: x \neq 1, x \neq -1, x \in R\}$   
 (c)  $\{x: x > 1, x \in R\}$
5. (a) 9 (b) -3 (c) -11 (d)  $4n - 3$  (e)  $4x^2 - 3$  (f) 4 (g) 4 (h) 4
9. (a) False (b) False (c) True (d) False (e) True (f) False
11. (a) 0 (b) 0 (c)  $4ab$  (d)  $2x_1 + h$

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1. (a)  $f(2)$  (b)  $x < 2$  (c) -12 3. even positive integers
5. (a)  $\{\frac{3}{2}, \frac{17}{2}\}$  (b)  $x \geq \frac{9}{2}$  7. (a) 100,000 (b) 99.835 (c) 102 (d) 0.06
9. (a)  $2a - 5$ ;  $2b - 5$ ;  $2a + 2b - 5$  (b) No 13.  $-t + 4$
15. (a)  $6x - 2$  (b)  $3x^2 - 2$  (c)  $6t + 1$